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Engineering Notes

Verification of *H* Flutter Analysis

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Nomenclature

C_m, C_g, C_k	=	coefficient of spline interpolation
d	=	damping ratio, $\frac{g}{k}$ or $\frac{\Re(p)}{\Im(p)}$
$E(g,k)$	=	kernel function
E^K, E^H, E^G	=	averaged error of k/p-k,H and g method
f	=	frequency, Hz, $\omega/2\pi$
g	=	reduced damping $\frac{\Re(p)l_{\mathrm{ref}}}{V}$
k	=	reduced frequency $\frac{\Im(p)l_{\mathrm{ref}}}{V}$
l_{ref}	=	reference length
L	=	number of mode shapes
N	=	number of support points
ω	=	circular frequency, rad/s, $\Im(p)$
p	=	differential operator (d/dt), also eigenvalue of the flutter equation
U	=	true air speed, m/s

Subscripts

i, j	=	mode number
m	=	support point number

I. Introduction

FLUTTER analysis is usually performed with basically two methods:

1) *k class*: the k method [1], which predicts the correct flutter instability;

2) *p-k class*: the p-k method [2,3], which predicts the damping and frequency trends fairly correct near zero damping.

The aforementioned methods are based on generalized aerodynamic forces obtained for purely oscillatory motions (g-invariant). When analyzing flutter diagrams, aeroelasticians are mostly dealing with the damping trend in the vicinity of zero damping ($g = 0$) while flight control system designers are also interested in damping trends in a larger g domain. The prediction of the damping and frequency trends can be further improved by methods belonging to the p-k class such as:

1) g: the g method [4], which improves the damping and frequency trends of the p-k method automatically near zero damping by taking into account the derivative of the generalized aerodynamic forces with respect to the damping at zero damping;

2) p: the p method [5], which improves the damping and frequency trends by taking into account the effect of nonzero damping by means of generalized aerodynamic forces, which are approximately valid for the damping-frequency area under consideration.

3) μ: The μ [6] method, which relies on fitting procedures to transform the aerodynamics to the state space.

However, methods (e.g., [7–9]) that estimate the aforementioned aerodynamic forces do hardly exist. In general analytical continuation of the generalized aerodynamic forces is applied. Approximation errors are accepted as a side effect due to the fitting procedures [10,11] associated with the generalized aerodynamic forces for purely oscillatory motion.

A novel flutter analysis method belonging to the p-k class was introduced and described [12]. In this method complex eigenvalue problems are solved for the damping and frequency at a set of predefined reduced frequencies k. The H method automatically extends the aerodynamic data obtained for purely oscillatory motions to damped and diverging oscillatory motions by means of a direct harmonic interpolation method, thereby improving the prediction of dampings and frequencies. The latter procedure was verified with respect to a subsonic oscillating diverging pitching flat plate [12]. In this article the H method is verified for the oscillating damped and diverging AGARD 445.6 wing in subsonic and supersonic flow and for an F-16 heavy store configuration in subsonic flow based on lifting surface aerodynamics. Results of the flutter analysis application will also be presented for the well-known AGARD flutter test case and the F-16 heavy store configuration. It should be noted that the p-k flutter method applied in this article is based on Hassig's method [2], and is not based on the British flutter method [3].

II. Direct Harmonic Interpolation Model

This section describes very shortly the interpolation/continuation method with respect to the generalized aerodynamic forces. To obtain the generalized aerodynamic forces for nonzero dampings, the generalized aerodynamic forces, which are computed for zero damping, have to be warped to the nonzero dampings space. Therefore, an interpolation is needed that provides implicitly the analytical continuation in the g, k space. Methods based on the class of spline techniques are used that are robust, automatic and cardinal. For a theoretical background on the spline techniques [13,14] should be consulted. Hounjet and Meijer [13] introduce the volume spline and various kernel functions and discusses their behavior and implementation aspects extensively. Reference [14] deals with recent developments.

Supposing the generalized aerodynamic forces GAF$(0, k_m)$ with respect to purely oscillating motions are calculated for N distinct frequencies k_m, we interpolate the data by:

$$\mathrm{GAF}\,(g,k) = C_0 + C_g g + C_k k + \sum_{m=1}^{N} C_m E(g,k;0,k_m) \qquad (1)$$

It is required that the interpolation is harmonic, meaning that the kernel function E satisfies the Laplace equation in a two dimensional spaced spanned by the reduced damping g and the reduced frequency k. The coefficients C are determined by satisfying the afore mentioned equation for GAF$(0, k_m)$ at the N support points m together with additional closure relations. Further details are presented in [12] including two types of kernel; the simple point kernel and the tentlike kernel. The tentlike kernel is:

$$E(g,k;0,k_m)^{\mathrm{tentlike}} = \int_{k_{m-1}}^{k_m} \frac{(k_{m-1}-y)\,\ell n((y-k)^2+g^2)}{k_{m-1}-k_m}\,\mathrm{d}y + \int_{k_m}^{k_{m+1}} \frac{(k_{m+1}-y)\,\ell n((y-k)^2+g^2)}{k_{m+1}-k_m}\,\mathrm{d}y \qquad (2)$$

which is evaluated analytically.

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III. Verification

To verify the direct harmonic interpolation method use is made of the general unsteady lifting surface method (GUL). The GUL doublet lattice (subsonic) and the GUL constant pressure (supersonic) method [15] are applied. The main application of the GUL is the prediction of steady and unsteady loads on configurations composed of thin wings in subsonic and supersonic linearized potential flow. The GUL has been developed to obtain a general unsteady lifting surface method with an almost unlimited application range with respect to Mach number, damping, frequency and configuration. The method has been verified for subsonic and supersonic flow by a comparison with similar methods, see also [16]. Further is has been shown in [8,17] that generalized airforces calculated for diverging motion with GUL can be accurately transformed to harmonic oscillatory motion. The important changes necessary for modeling damped an diverging oscillatory motion are presented in [17].

The following numerical models are embedded in the GUL code:

1) The potential gradient (PG) method based on a series development of the elementary doublet function as introduced by Jones and Appa [18] for supersonic flows.

2) The improved PG method [19] by introducing the elementary pressure doublet function for dealing with wakes.

3) The constant pressure panel method reported in [15] for supersonic flows aiming at high reduced frequencies and low supersonic Mach numbers with a minimum number of required panels. A similar method has been developed earlier by Appa and Smith [20].

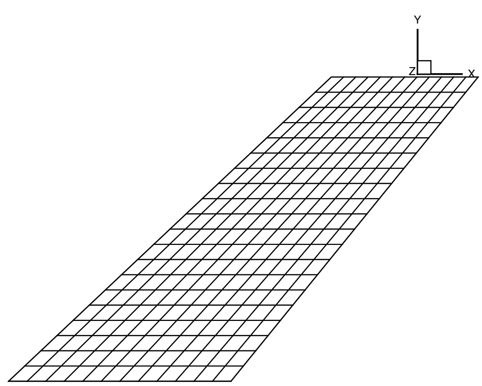

Fig. 1 Panelling of AGARD wing 445.6.

4) The doublet lattice method for subsonic flow based on the formulations of Rodden et al. [21]

The verification is conducted for the 3-D AGARD standard aeroelastic configuration in subsonic and supersonic flow and an F-16 heavy store configuration in subsonic flow. The AGARD configuration (Model weakened no. 3) is described in [22].

A. AGARD Wing

The GUL method is first applied for 3 Mach numbers 0.5, 0.96, and 1.14 using a 20×12 panel discretization as depicted in Fig. 1 for 4 vibration modes and for 41×21 reduced frequencies in the range $g = -2..2$ and $k = 0..2$ with a step size of 0.10. The selected ranges are typical for aero elastic studies and the l_{ref} was 1.8 times the root chord of the wing. Next the data for $g = 0$ is used by the aforementioned direct harmonic interpolation method and warped to $g \neq 0$ with the tentlike kernel.

Results obtained with the H method are compared with results obtained with the k/p-k and g methods. The k/p-k methods assume that the generalized forces are invariant with respect to g. The g method assumes $\mathrm{GAF}(g,k) = \mathrm{GAF}(0,k) + g\frac{\partial \mathrm{GAF}(0,k)}{\partial k}$. The comparison is based on the averaged relative errors defined as:

$$E^H(g,k) = \frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L}\left|\frac{\mathrm{GAF}^H_{ij}(g,k) - \mathrm{GAF}^{\mathrm{GUL}}_{ij}(g,k)}{\mathrm{GAF}^{\mathrm{GUL}}_{ij}(g,k)}\right| \quad (3)$$

$$E^K(g,k) = \frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L}\left|\frac{\mathrm{GAF}^K_{ij}(g,k) - \mathrm{GAF}^{\mathrm{GUL}}_{ij}(g,k)}{\mathrm{GAF}^{\mathrm{GUL}}_{ij}(g,k)}\right| \quad (4)$$

$$E^G(g,k) = \frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L}\left|\frac{\mathrm{GAF}^G_{ij}(g,k) - \mathrm{GAF}^{\mathrm{GUL}}_{ij}(g,k)}{\mathrm{GAF}^{\mathrm{GUL}}_{ij}(g,k)}\right| \quad (5)$$

with $L = 4$. E^H, E^K and E^G denote the error of the H, k/p-k and g method, respectively. GAF^H_{ij}, GAF^K_{ij}, GAF^G_{ij}, and $\mathrm{GAF}^{\mathrm{GUL}}_{ij}$ denote the generalized air forces directed along mode i due to deformation along mode j resulting from the H, k/p-k, g method, and the GUL method, respectively.

Figures 2–4 show contour plots of the averaged relative error in percentages of the real part and the imaginary part for Mach = 0.5, Mach = 0.96, and Mach = 1.14, respectively. First it should be remarked that the GAFs obtained for the damped oscillatory motion are prone to numerical error due to the association with exponential growing functions with exponent equivalent to $\frac{1}{1-\mathrm{Mach}}$. This is

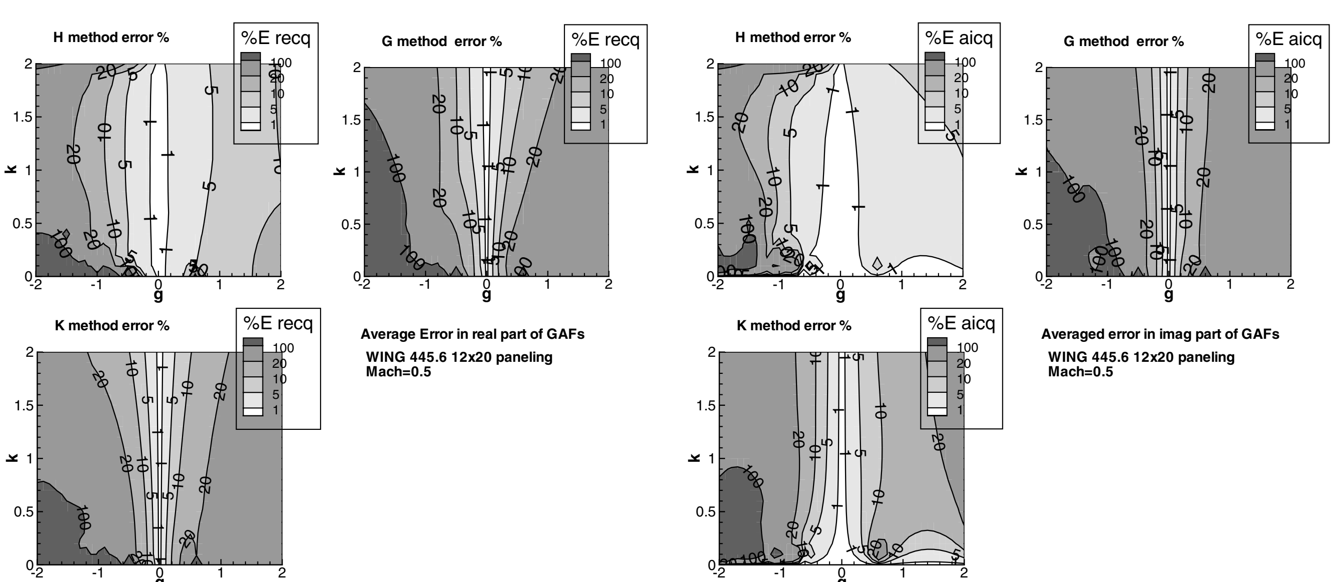

Fig. 2 Real and imaginary part of averaged relative % errors in E^H, E^K, and E^G of the AGARD wing 445.6 at Mach = 0.5.

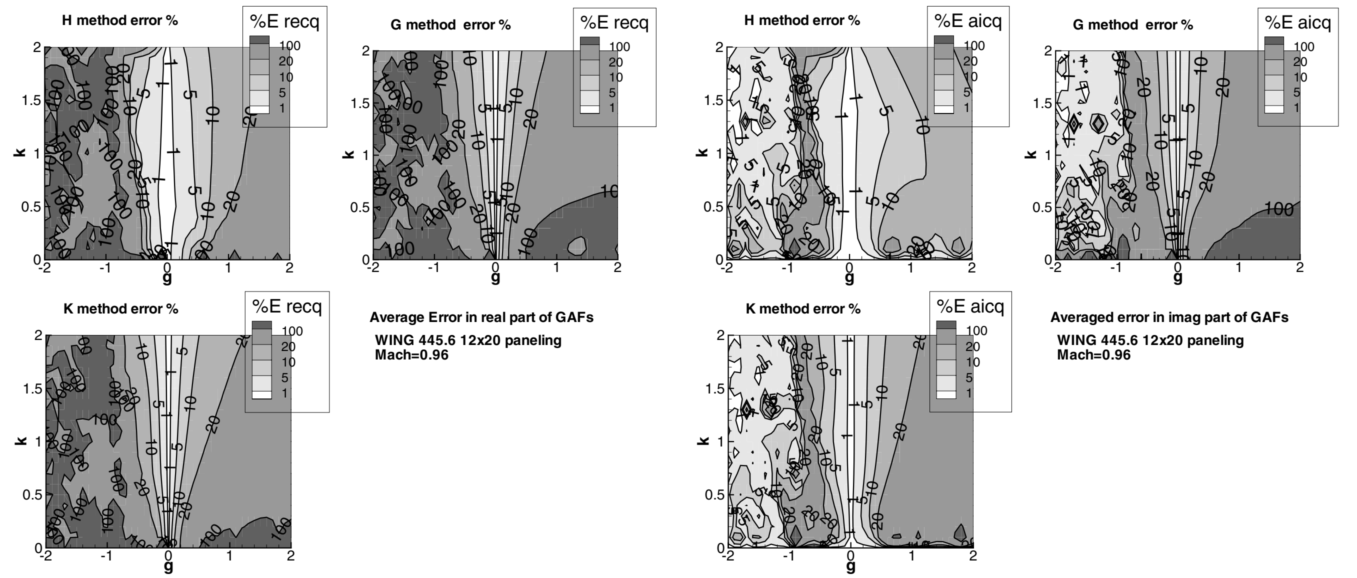

Fig. 3 Real and imaginary part of averaged relative % errors in E^H, E^K, and E^G of the AGARD wing 445.6 at Mach = 0.96.

particular demonstrated in Fig. 3 for Mach = 0.96 where the results for g-values smaller than -1 are chaotic. This behavior can be reduced by using more panels and/or by improving integration. At reduced dampings approaching zero the error is very small for all methods and the conclusion can be drawn that the transition to nonzero damping of the generalized air forces is smooth. Considering the whole domain the H method is preferable, followed by the k/p-k method and last is the g method. Both latter approaches show larger error levels compared with the error levels observed for the H method. For positive g the errors are smaller compared with negative g. At Mach = 0.5 the error is minimal.

The chaotic behavior for negative g observed is further illustrated in Fig. 5, which shows a contour plot of the relative error in the real part and imaginary of GAF_{11}^{H}, GAF_{11}^{K}, and GAF_{11}^{G}. In addition, the original $\mathrm{GAF}_{11}^{\mathrm{GUL}}$ is compared with GAF_{11}^{H} for Mach = 0.96. Clearly the directly calculated GUL data are not reliable for g-values below -0.8 at this transonic Mach number.

B. F-16 Heavy Store Configuration

The GUL method is further applied for Mach = 0.9 using the panelling depicted in Fig. 6 for 13 vibration modes and for 41×21 reduced frequencies in the range $g = -2..2$ and $k = 0..2$ with a step size of 0.10. The l_{ref} was 0.50 times the root chord of the wing. Next the data for $g = 0$ is used by the direct harmonic interpolation method and warped to $g \neq 0$ with the tentlike kernel. The panelling is typical for standard flutter analysis performed for the F-16.

Figure 7 shows a comparison of the original GUL data to the warped data in terms of contour plots of the averaged relative error in percentages of the real part and the imaginary part for Mach = 0.9. The error in the imaginary part is smaller compared with the error in the real part for all methods. Again it is observed that the GAFs obtained for the damped oscillatory motion for g-values smaller than -0.5 are chaotic. The same remarks can be made as earlier made for the AGARD wing. In general the k/p-k method and the g method show larger error levels compared with the error levels observed for the H method. For positive g the agreement is better compared with negative g. The error level is higher as the one observed for the AGARD wing. This might be probably due to the rather coarse panelling and/or the fact that all 13 modes contribute to the error.

IV. Flutter Applications

Flutter analysis is performed on the basis of matching altitude, Mach number, density, and speed in the standard U.S. atmosphere, which leads to consistent data.

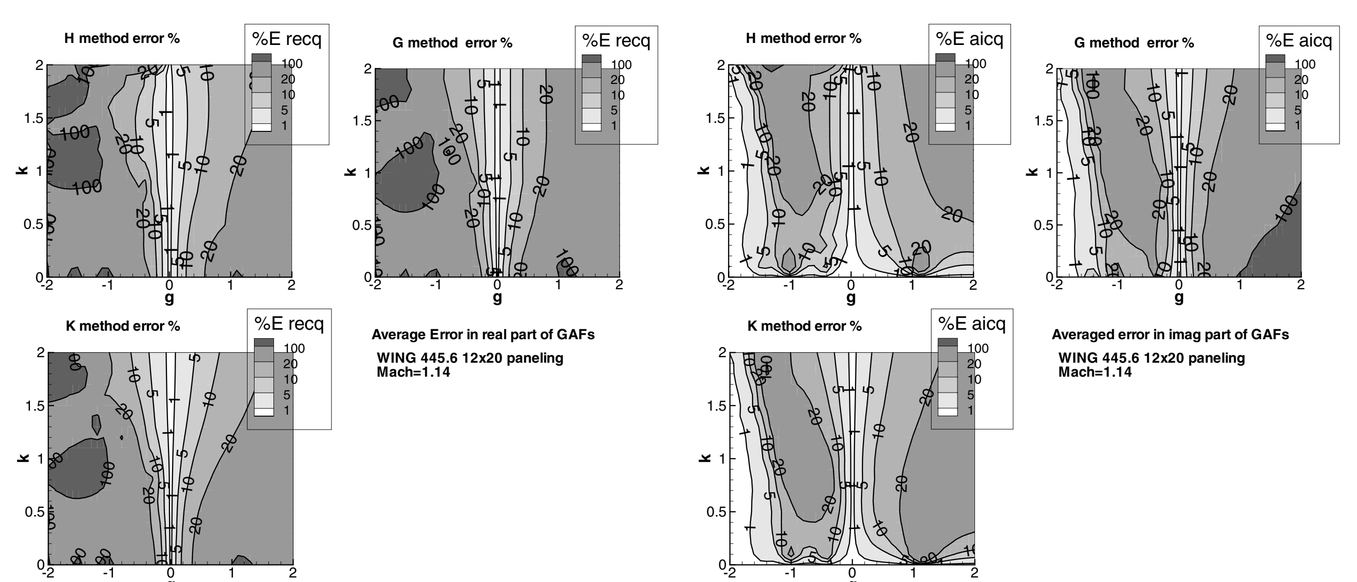

Fig. 4 Real and imaginary part of averaged relative % errors in E^H, E^K, and E^G of the AGARD wing 445.6 at Mach = 1.14.

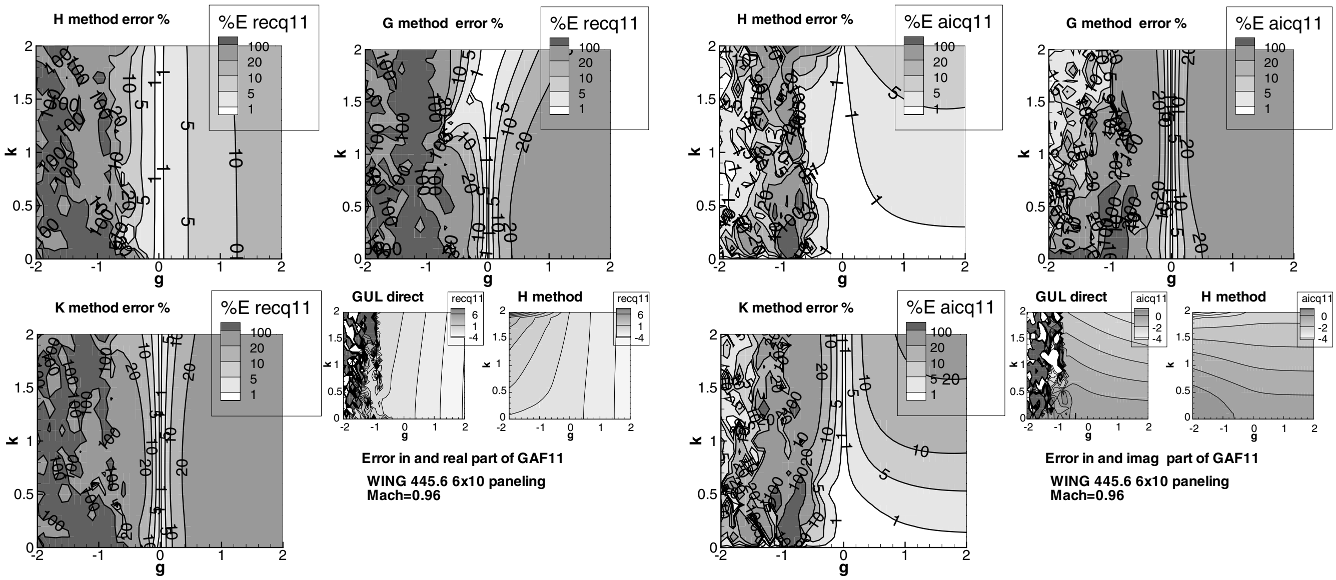

Fig. 5 Real and imaginary part of averaged relative % errors and real and imaginary part of GAF_{11}^{GUL} and GAF_{11}^{H} of the AGARD wing 445.6 at Mach = 0.96.

Results of the H-method for the AGARD wing at Mach number 0.5, 0.96, and 1.14 are compared in Fig. 8 with results obtained with the p-k method. The figure shows plots of damping ratio d, frequency f and reduced damping g versus equivalent airspeed. Both methods predict the same flutter speed and instability mechanism. The dampings and frequencies of both methods agree up to high levels of the dampings and velocity. The H method seems to reduce the damping levels at relatively high negative damping and speed and increases slightly the damping levels at relatively high positive damping and speed. With increasing Mach number the damping levels are reduced. The k values not plotted here are well below two and the g range values that really should matter are well within the 5% error border of Figs. 2–4.

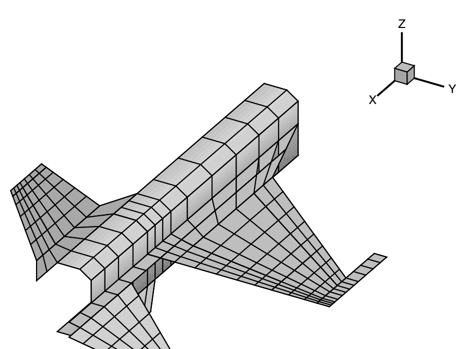

Fig. 6 Panelling of F-16.

Results of the H-method for the F-16 heavy store configuration at Mach number 0.9 are compared in Fig. 9 with results obtained with the p-k method. The figure shows plots of damping ratio d, reduced frequency k, and reduced damping g versus equivalent airspeed (VEAS). Again both methods predict the same flutter instability mechanism. The dampings and frequencies of both methods agree up to very high levels of the dampings and velocity. The H method seems to affect only a single mode after the flutter point has been passed. The H method seems to reduce the damping levels at relatively high negative damping and speed. A tiny bit more damping can be noticed with respect to the unstable mode. The k values are well below two and the g range values that really should matter are

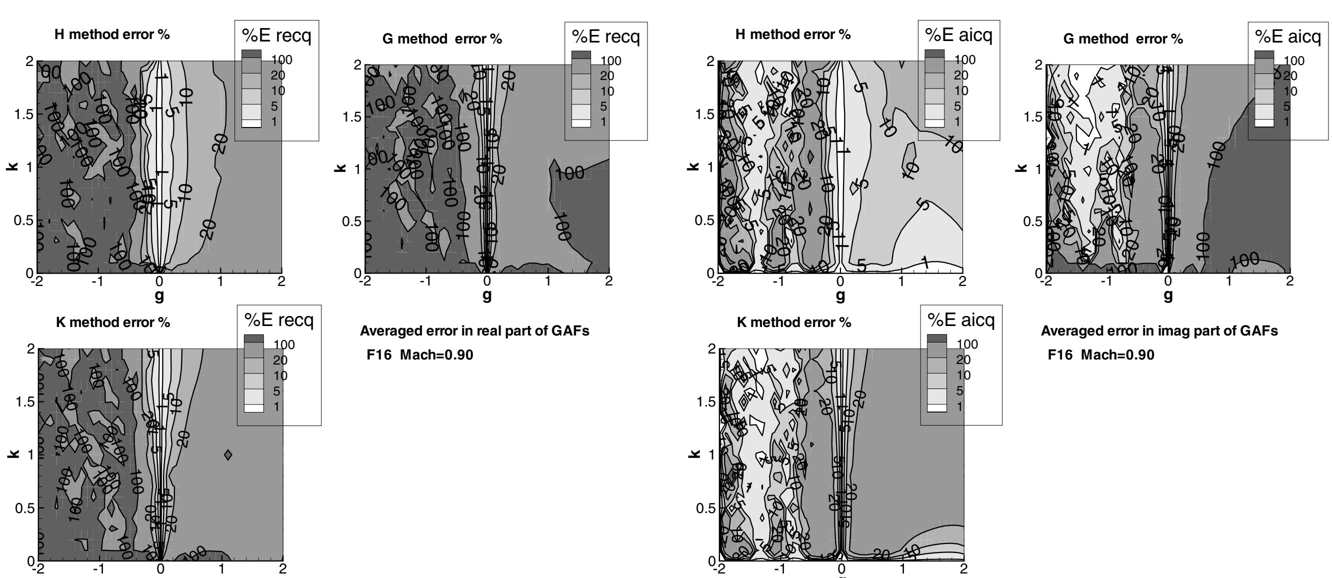

Fig. 7 Relative % error in real and imaginary part of GAF_s for F-16 at Mach = 0.9.

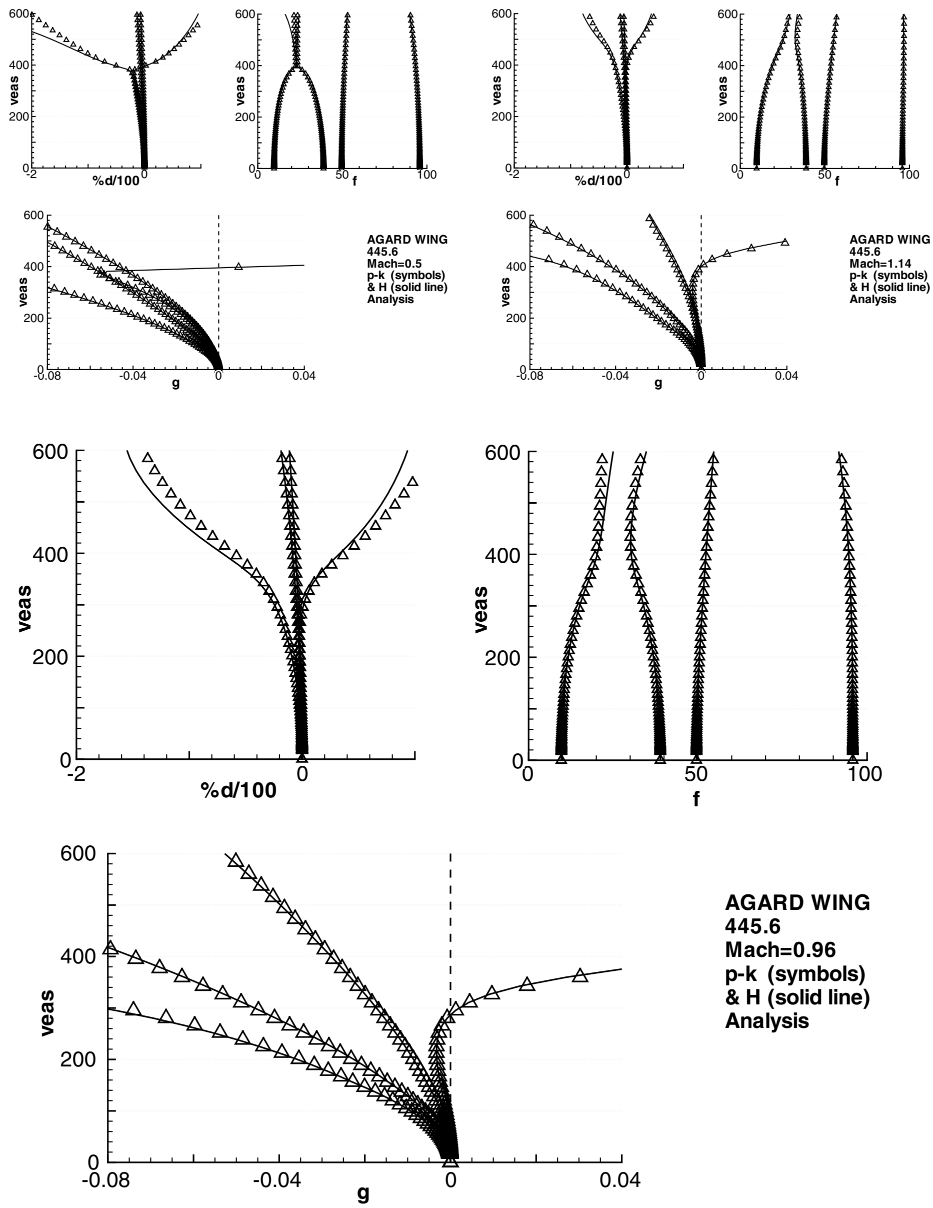

Fig. 8 Flutter diagrams for the AGARD wing 445.6 at Mach number 0,50,0.96 and 1.14 using the *p-k* method (symbols) and the *H*-method (solid).

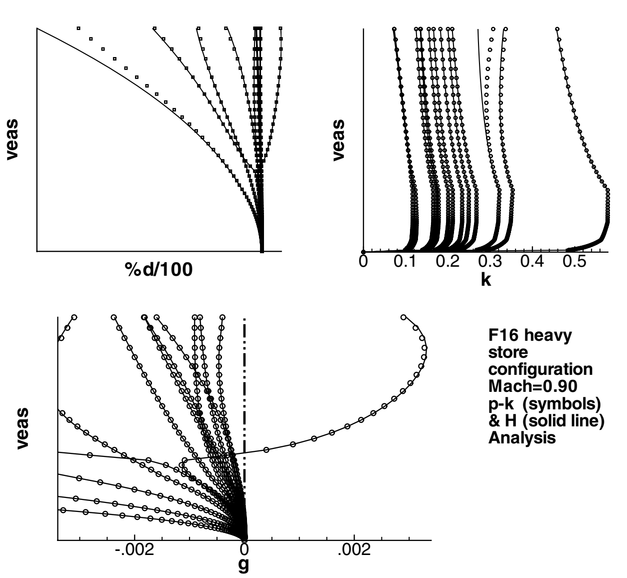

Fig. 9 Flutter diagrams for the F16 Heavy store configuration at Mach number 0.90 using the *p-k* method (symbols) and the *H*-method (solid).

well within the 5% error border of Fig. 7. The kink in the *k*-VEAS plot is due to the fact that the standard US atmosphere model is continued below zero altitude.

The presented flutter analysis highlights again the fact that results of the *p-k* methods are fairly correct beyond zero damping using lifting surface aerodynamics. This is due to the smooth damping behavior of this type of generalized air forces near zero damping. However, in transonic flow the damping behavior near zero damping of the generalized airforces can be very different as shown in [8], and might have a larger effect.

V. Conclusions

The *H* method is verified for the AGARD aeroelastic test case and the F-16 heavy store case using lifting surface aerodynamics for unsteady subsonic and supersonic flow. The *H* method contains a simple procedure that automatically extents the aerodynamic forces data obtained for purely oscillatory motions to damped and diverging oscillatory motions by means of a straightforward fitting free interpolation. The fitting free interpolation is verified successfully for both configurations and is superior to the classical *g*-invariant method and the so-called *g* method. Also as the accurate estimation of aerodynamic force data for nonzero *g* is not much practised it is advisable to use the H method in situations where accurate aerodynamic damping estimates are relevant.

The differences between the H and p-k flutter analysis of the AGARD wing and the heavy store F-16 configuration are very small at low levels of dampings. A different damping trend capturing at high levels of dampings is observed in comparison to the p-k method. It needs to be investigated if the effects are more relevant for modern aircraft and for aerodynamic forces generated by modern unsteady aerodynamic methods. This procedure may assist the aeroelastician in making improved estimates of aerodynamic dampings at $g < 0$ conditions to support flight flutter testing and probably offers potential for flight control system design/analysis.

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